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Mathematical Knowledge for Teaching Developmental Courses at the Community College: An Unexplored Terrain

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ABSTRACT

The domain of Mathematical Knowledge for Teaching (MKT) has provided theoretical and empirical advancements to teacher education scholarship. Today, MKT is widely recognized as a critical component of teachers' preparation, professional development, and experience. Early studies at the elementary level set the stage for defining, measuring, assessing, and augmenting this knowledge in pre-service and in-service teachers. Subsequent studies are demonstrating MKT's significance in university and secondary teaching. Despite MKT's growing influence in education, one institution that has been largely ignored is the community college. Given the community college's role of remediation alongside the diverse and often underprepared student body it serves, we urge the mathematics education community to examine MKT in the context of community college developmental mathematics. Based on preliminary interviews with experienced faculty at two-year institutions (n = 12), we propose a provisional amendment to the domains of MKT – the Caring Map. We speculate that in order to make sense of developmental mathematics teaching at the community college, research is needed to determine the extent to which teacher caring supports mathematics teaching, student learning, and student achievement.

Educational researchers will recognize Mathematical Knowledge for Teaching (MKT) as a well-developed, evolving area of scholarship. Today, we know much about the special craft that is *mathematics teaching* – what knowledge teachers bring to the classroom (Ball, Hill, & Bass, 2005; Ball, Lubienski, & Mewborn, 2001; Hill, Sleep, Lewis, & Ball, 2007), how teachers develop, hold, and convey this knowledge (Ball, Thames, & Phelps, 2008; Carpenter, Fennema, Peterson, & Carey, 1988; Eisenhart et al., 1993; Even, 1993; Silverman & Thompson, 2008; Simon, 1993; Thompson, 2016), how to measure and assess this knowledge (Hill, Ball, & Schilling, 2008; Hill, Rowan, & Ball, 2005; Hill, Schilling, & Ball, 2004; Hill et al., 2007), how students are impacted by this knowledge (Hill et al., 2005), and how knowledge can be developed and augmented through professional development and/or coaching (Ghousseini, 2017; Hill & Ball, 2004; Philipp et al., 2007). Developments in the field are ongoing; new pieces of the puzzle continue to illuminate the existing fund of knowledge, and questions abound.

Despite the above contributions in K-12 and university settings, there is a particular arena that has been largely ignored: the community college (Nabb & Murawska, 2016, 2018, in press). Zooming in further, *developmental mathematics teaching* at the community college has been mostly excluded from formal study. On the one hand, the history of mathematics education research makes this understandable. For example, deficiencies in elementary teachers' content knowledge led to a surge of studies in the 1990s motivated by concerns for those guiding our most vulnerable and malleable resources – young children. In more recent years, interest in secondary-level teaching (Howell, Lai, Phelps, & Croft, 2016; McCrory, Floden, Ferrini-

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Mundy, Reckase, & Senk, 2012; Speer, King, & Howell, 2015) and post-secondary/university teaching (Khakbaz, 2016; Rasmussen & Marrongelle, 2006; Speer et al., 2015; Speer, Smith, & Horvath, 2010; Wagner, Speer, & Rossa, 2007) have led to analogous investigations of teacher knowledge. Additionally, public perceptions attest to the stigma that is often associated with the community college – institutions labeled as less than that of universities and colleges (Clark, 1960, 1980; Goldrick-Rab, 2010; Jennings, 1970). Given the above markers, the lack of research attention on community college mathematics teaching is hardly surprising.

On the other hand, we find the above omission both perplexing and inexcusable; Goldrick-Rab (2010, p. 449) has called it "remarkable." After all, community colleges are institutions whose primary concern is *quality teaching*. Faculty at community colleges enter the profession because they are – or hope to become – skilled and caring teachers. So why has MKT been ignored in a setting where it seems most plausible to study? How can we consider community colleges as democratizing education, when, in fact, research into teaching practices is so slim? Moreover, while developmental mathematics enrollments begin to decline, why do completion rates in traditional developmental sequences remain low[1] (Bailey, 2009; Boylan, 2011)? While some students do succeed, we remain in the dark about *what teachers do* and *how they do it*. Given the recent reform movements endorsed by the American Mathematical Association of Two-Year Colleges (AMATYC) (e.g., *Carnegie Pathways, Mathways* and *New Life*) as well as the critical updates to AMATYC's standards documents (AMATYC, 2018), the time is ripe to examine MKT in community college mathematics. We see this call as especially urgent at the developmental level.

To be clear, prior to many of the studies of pedagogical content knowledge (PCK) from the 1990s, many asked the question, "What's 'sophisticated' about elementary school mathematics?" (Wu, 2009, p. 7). It is long overdue we ask the similar question: *What is sophisticated about teaching developmental mathematics in the community college?*

In this article, we have several objectives. First, we review the literature on Mathematical Knowledge for Teaching (MKT). Much of the research points to examples of the types of knowledge that skilled K-12 and university teachers are purported to possess and use in their respective environments. By doing so, we propose MKT in community college developmental mathematics as a setting worthy of investigation.[2] Next, we glean information from interviews conducted with 12 expert community college mathematics faculty. From here, we suggest a provisional amendment to existing models of MKT that reflects *community college mathematics instruction* and the *students* it serves. The work here, we believe, is important to upholding the community college's promise of educating all individuals and advancing social mobility.

Developmental mathematics is currently undergoing a national facelift in order to provide meaningful experiences for students and to ensure higher degrees of success (Burdman, 2018). However, what the next generation of community college mathematics teachers needs to know to make this happen is mostly a blank slate. Our charge is to provide some guidance on what *good teaching* at this level can and should look like. This article provides a small first step in this direction and mirrors the work done by Ball, Hill and colleagues at the elementary level (e.g., Hill et al., 2005; Hill, Ball, et al., 2008), Thompson and colleagues at the secondary level (e.g., Silverman & Thompson, 2008; Thompson, 2016), and Speer and colleagues at the university level (Speer et al., 2015). The time is now. Improving the state of developmental education in the United States can and should be in everyone's best interest and the community college provides the means to carry out this important work.

Literature review

Mathematical knowledge for teaching

It is well-documented in the literature that effective mathematics instruction is predicated not only on the instructor's knowledge of mathematics and pedagogy, but specifically on the instructor's *mathematical knowledge for teaching* (MKT) (e.g., Ball et al., 2008; Hill et al., 2005; Hoover, Mosvold, Ball, & Lai, 2016; Selling, Garcia, & Ball, 2016). This type of knowledge goes beyond the mathematics content itself, as it is the interplay of mathematics *and* pedagogy.

This special form of teacher knowledge described by Shulman (1986) is "*pedagogical content knowledge*: subject knowledge *for* teaching" (p. 9). Shulman's (1986) seminal paper on pedagogical content knowledge (PCK) brought about a revolution in the world of mathematics education, resulting in an explosion of research that continues to define, analyze, and assess PCK. Shulman explains that PCK is still subject matter knowledge, but it is subject matter knowledge explicitly needed for teaching – the ways teachers formulate and represent content in order to make it accessible and comprehensible to others:

> Mere content knowledge is likely to be as useless pedagogically as content-free skill. But to blend properly the two aspects of a teacher's capacities requires that we pay as much attention to the content aspects of teaching as we have recently devoted to the elements of teaching process. (Shulman, 1986, p. 8)

In the last thirty years, a powerful research line has emerged that has shaped the fields of mathematics education and teacher education.

In this section, we will describe how the research on mathematical knowledge of teaching has evolved, from its kernel at the elementary level, to its expansion into both secondary and university mathematics education. Although there exist some parallels, mathematical knowledge of teaching has not been adequately researched at the community college level.

Search methodology

We searched the educational databases ERIC and Education Research Complete to obtain relevant research literature on the topic of mathematical knowledge of teaching. At first, we searched for journal articles whose abstracts contained the key word *mathematical knowledge for teaching* or *MKT* from the year 2000 to present day to ensure we captured the most recent trends in the field. We then searched subsets of MKT research by instructional level. Table 1 provides the reader with details.

As the reader can see, we found no journal articles in either database whose abstracts contained explicit references to mathematical knowledge for teaching in community colleges. Subsequently, we expanded our search to include any article that focused on the *practice* of teaching mathematics at the community college level, including journal-specific searches, the details of which we will articulate at the end of this section. But first, we begin by describing the evolution of the study of mathematical knowledge of teaching.

Evolution of mathematical knowledge for teaching in elementary mathematics

Like many mathematics education researchers have previously documented, we recognize the teaching of mathematics as a complex, multifaceted activity. The body of research that has provided our foundational

Table 1. Number of journal articles on mathematical knowledge of teaching, 2000–2017.

Key Words in Abstract		Database	
Appeared in Abstract	Search Narrowed by Instructional Level	ERIC	Education Research Complete
"mathematical knowledge for teaching" or "MKT"	–	146	129
"mathematical knowledge for teaching" or "MKT"	"elementary" or "k-5"	34	28
"mathematical knowledge for teaching" or "MKT"	"secondary" or "high school"	17	15
"mathematical knowledge for teaching" or "MKT"	"university" or "college" or "higher education"	14	11
"mathematical knowledge for teaching" or "MKT"	"community college" or "two-year college"	0	0

understanding of MKT is the work of Ball and colleagues (e.g., Ball et al., 2008; Hill et al., 2005, 2007). Their work undergirds the movement that is continuing to inform the evolution and progression of MKT.

Early MKT research unpacked mathematical knowledge for teaching elementary school mathematics by focusing on the *enactment* of teaching – that is, by examining what teachers *do* in the classroom. These enactments include how teachers design tasks, facilitate discussions, explain concepts, and challenge student misconceptions. This was in contrast to much of the *production function* research literature prior to this time, which typically measured teacher knowledge with proxy variables such as college degree, years of experience, courses completed, certifications, classroom characteristics, student/teacher ratio, and curriculum materials.

During this time, many scholars (e.g., Ball, 1988; Ball et al., 2001) asserted that knowledge of mathematics is necessary but insufficient to teach mathematics. The key to student learning is the teacher's mathematical knowledge for teaching, though the researchers acknowledge that its nature and role was still unclear. Because knowing one's MKT is not equivalent to the enactment of this MKT, researchers continued to focus on teacher practice to unmask MKT. For instance, Hill et al. (2005) studied elementary teachers' MKT and found that the knowledge quality teachers *use* – not the knowledge they *possess* – was a more accurate prediction of quality teaching, as evidenced by increased student achievement on K-8 standardized test scores.

Subsequent work was done to measure and assess MKT. Hill et al. (2004) demonstrated that MKT is indeed multidimensional, including knowledge specific to topic (e.g., number, algebra) and domain (knowledge of content, students). Through this process, they provided evidence of the subdomain *specialized content knowledge* (SCK) for teaching. Hill et al. (2007) continued to explore the measurement and assessment of MKT through observations of teacher practice, mathematical interviews, and analysis of paper-and-pencil tasks. Then Hill, Ball, et al. (2008), Manizade and Mason (2011), Phelps and Howell (2016), and Selling et al. (2016) all contributed to the research literature with their analyses of multiple-choice assessments, design of assessment tools, analysis of assessment tasks, and the development of an MKT assessment framework, respectively.

To further delineate MKT, Ball et al. (2008) used an empirical approach to examine the work of teaching; the researchers analyzed video recordings of the day-to-day practice of teaching, and formulated a practice-oriented perspective of MKT. Inherent in this methodology was ensuring that the measures of mathematical knowledge for teaching were designed to "situate knowledge in the context of its use" (*p*. 403). Their result was a refinement of Shulman's (1986) definitions of

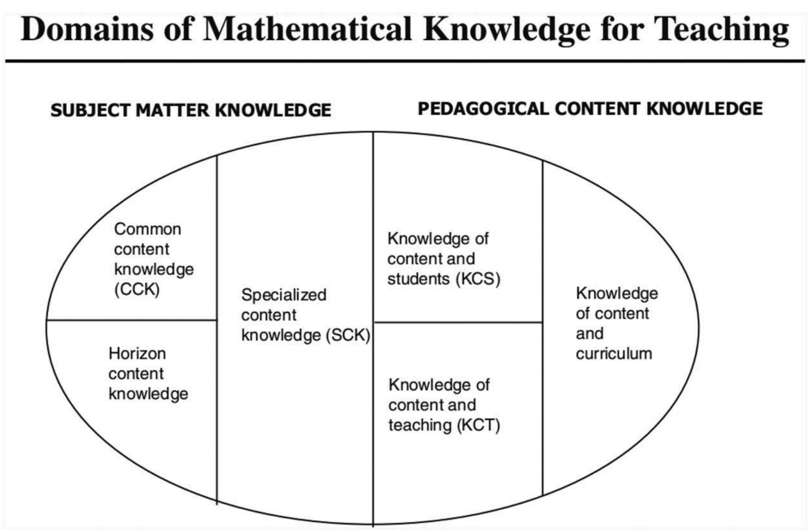

Figure 1. Ball et al. (2008) domains of MKT.

subject matter knowledge and pedagogical knowledge, in which the researchers entitled the Sub-Domains of Mathematical Knowledge for Teaching, illustrated in Figure 1.

Affectionately known as the egg, these domains further clarify what constitutes MKT and continue to inform new research studies. The six subdomains are defined by Ball et al. (2008) as follows:

Subject Matter Knowledge Subdomains

- *Common content knowledge* (CCK): "mathematical knowledge and skill used in settings other than teaching," (*p*. 399) or "mathematical knowledge known in common with others who know and use mathematics" (Ball et al., 2008, p. 403)
- *Horizon content knowledge* (HCK): an "awareness of how mathematical topics are related over the span of mathematics included in the curriculum" (Ball et al., 2008, p. 403)
- *Specialized content knowledge* (SCK): "mathematical knowledge and skill unique to teaching" (Ball et al., 2008, p. 400)

Pedagogical Content Knowledge Subdomains

- *Knowledge of content and students* (KCS): "knowledge that combines knowing about students and knowing about mathematics" (Ball et al., 2008, p. 401)
- *Knowledge of content and teaching* (KCT): knowledge that "combines knowing about teaching and knowing about mathematics" (Ball et al., 2008, p. 401)
- *Knowledge of content and curriculum* (KCC): knowledge that combines knowing about curriculum and knowing about mathematics (Ball et al., 2008)

Of special interest to Ball et al. (2008) was specialized content knowledge (SCK), as they hypothesized that this subdomain was the key to effective teaching and learning. Note that SCK is a subdomain of subject matter knowledge, but it does not necessarily intertwine with knowledge of students or knowledge of teaching, and it is not usually familiar to mathematicians or educated adults. However, researchers acknowledge that the boundaries between many of the subdomains remain fuzzy. For instance, when a teacher accurately analyzes a student's error, the teacher may be drawing from SCK or KCS, depending on that teacher's mathematical knowledge and past experience with students in similar situations.

In summary, MKT was born out of the observation and study of elementary school teacher practice, yet it was recognized that the MKT model could have implications for other levels of mathematics as well. In fact, according to a meta-analysis of empirical MKT-related research literature from 2006 to 2013, the majority of MKT research focused on teachers of elementary school (K-8) or middle school (5–9) with few studies focused on the secondary level (Hoover et al., 2016). Surprisingly, these authors found just *three* empirical studies that focused on MKT at the tertiary level. This led us to investigate the extent to which MKT is studied at levels other than elementary – would MKT look different in the secondary, community college, or university settings?

MKT-related research in secondary mathematics

Many studies sought to use Ball (1988, 2008) framework to conceptualize MKT at the secondary level. Of these studies, many narrow their focus to a particular mathematical topic. For instance, Even (1993) found that even if preservice secondary teachers' subject matter knowledge (SMK) of functions was adequate, s/he may choose not to apply this knowledge in the classroom (i.e., exhibit PCK). Others focused on developing frameworks for investigating and assessing MKT in algebra (e.g., Hill, Blunk, et al., 2008; Hill et al., 2008; McCrory et al., 2012).

Interestingly, the work of some scholars is closely related to MKT, though the authors may not label it as such. For example, Thompson's (2016) study examined high school teachers' *mathematical* *meanings* by investigating their decisions and actions to describe the rationale behind their classroom behaviors. While Thompson's work is related to that of Ball and colleagues in its empirical approach, Thompson makes the distinction that effective teaching requires critical reflection over time. Similarly, in Silverman and Thompson's (2008) study, rather than focusing on the *knowledge* a teacher must have to effectively teach, the researchers examined the *understandings* that teachers develop through mathematical experiences and reflection; these understandings, acquired through the years, serve as the main vehicle for improving mathematical instruction.

What is the chief difference in these perspectives? The former (e.g., Ball et al., 2008) suggests that this knowledge base is finite and once known, we can train teachers in such core knowledge. The latter (e.g., Thompson, 2016) hints that the process is developmental (i.e., developing one's professional practice, adapting in practice, and continuing so indefinitely) and that several essential understandings influence teaching more broadly. Either way, the philosophy that effective teachers develop an entirely new form of knowledge through experience and reflection is important to consider when framing new research to investigate MKT (e.g., see Philipp et al., 2007).

Overall, researchers have found that the elementary and middle level MKT work of Ball and colleagues can be generalized to secondary mathematics teaching. But can the MKT egg be applied to university or community college mathematics?

MKT-related research in university mathematics

As a result of the review of the literature, our overarching assumption is that the nature of MKT is potentially unique for college and university level teaching. In other words, generalizing from one arena (i.e., K-12) to another is not a simple or necessarily valid step (Speer et al., 2015, 2010). In this section, we present the research literature which speaks to the potential similarities and differences of examining MKT in higher education.

One commonality found among the results of university MKT studies was the importance of sociocultural factors in the classroom; we highlight the results of two such studies. For example, Rasmussen and Marrongelle (2006) investigated instruction and discourse in a university differential equations course in which the teacher implemented inquiry-based learning. Particularly useful pieces in their findings may inform subsequent research in community college settings: studying enacted expertise in mathematics teaching and the importance of social and socio-mathematical norms (Yackel & Cobb, 1996) while enacting MKT.

A second study that examined classroom discourse in a university setting was the case study by Tsay, Judd, Hauk, and Davis (2011) that focused on classroom discourse and the shaping of one college algebra professor's PCK. The results underscore how attention to students' sense-making during classroom discussions in the context of social, cultural, and behavioral norms can help university instructors thus improve their practice. Because the researchers were interested in how the instructor turned "teacher intentions into actions" (Tsay et al., 2011, p. 209) – a common theme found in the works of Ball and Thompson – we find their results particularly relevant.

As we attempt to forecast how MKT could be conceptualized in community college developmental courses, the research by Speer and colleagues has proven to be most promising. For instance, Speer et al. (2015) examined whether MKT, as it is presently known, along with its subdomains – especially CCK and SCK – are transferable to secondary and college level instruction. The root of this study is based on the facts that (a) very few studies have examined MKT in secondary and post-secondary settings, and (b) MKT is contextually bound by the settings/population in which it is examined (i.e., elementary school). Hence, Speer et al. question transferability and generalizability to different grade levels.

For instance, what is definitively known is (1) the knowledge needed to *teach* mathematics (assumption: at the elementary level) is different from the knowledge of most mathematically-educated adults, and (2) the knowledge in (1) is likely not learned in traditional college math courses (e.g., calculus, linear algebra). For elementary level mathematics instruction, the distinctions between CCK and SCK are well known. However, consider the discussion of a combinatorics problem in a secondary classroom, where students discover an interesting way to solve a problem using the Fibonacci sequence (Speer et al., 2015). To those individuals with significant training in mathematics, this would be characterized as CCK. However, the teacher needs to decide – among other things – should this solution be shared with the class? How does it connect to the conventional solution and/or the goals of instruction? Also, is this *really* the Fibonacci sequence or something closely related? The bottom line here is that content knowledge and PCK intermingle. So perhaps the SCK conceptualization – i.e., the definition used in the literature by Ball et al. (2005) – does not have an analogous counterpart in university mathematics, as the present definition is too closely an artifact of elementary level instruction.

Other studies by Speer have implications for future research in MKT as well. For example, Wagner et al. (2007) studied the teaching practices of a mathematician implementing an inquiry-oriented differential equations course, and results included descriptions of how instructors acquire knowledge that is *separate* from CCK, such as the capacity to coordinate student responses during classroom discussions, and the ability to predict students' misconceptions. Similarly, Speer and Wagner (2009) focused on the knowledge that a mathematician would need to effectively lead whole-class discussions. The researchers acknowledged that the strong content knowledge mathematicians bring to the table is beneficial, but they also articulated the mathematician's challenges in orchestrating classroom discussions (e.g., the ability to build upon students' intuitive and sometimes surprising thinking strategies, determining next steps when a student's contributions are not mathematically valid, etc.).

In recent developments, MKT research is making its way into university teacher preparation programs. A recent study by Ghousseini (2017) addresses MKT not from afar but from within the practice of teaching: How do novice teachers enrolled in a master's program embed learned knowledge (MKT) in the active work of elementary mathematics instruction and how can they be further supported to do so by teacher educators? The author argues that this practice-based perspective supports teachers in their development of MKT.

In summary, MKT may look different at the secondary and university levels precisely because the populations of students, the characteristics of the instructors, the goals of instruction, the nature of the content, and the culture of the institutions, are unique to each setting. Consequently, the constructs of MKT may need to be modified as researchers seek to extend the trajectory into different arenas such as the community college.

Where is MKT research in community college?

Why is MKT not studied at the community college? To begin to answer this question, we expanded our search to include journal articles that investigated the *practice* of mathematics teaching, with broader key words. We also searched within specific journals to ensure that we were not missing any salient research. Table 2 provides the number of journal articles published in 2000–2017 that focus on community college mathematics. Still, this search of community college and developmental education research literature produced very few empirical articles that investigated instructional practices.

In our quest for community college research, we searched the *Community College Journal of Research and Practice*, and though we found 51 articles that listed *mathematics* in the abstract, only a handful articulated anything about mathematics *instruction*, and none of these articles were on MKT. Similarly, we searched *New Directions for Community Colleges* and the *Journal of Developmental Education*, and found many articles that focused on mathematics, but again, virtually no empirical articles on mathematics *instruction*; instead, the articles included proxy variables such as state and institutional policies, placement test procedures, program evaluation, and curriculum pathways. These search results are analogous to the findings of Mesa, Wladis, and Watkins (2014), who stated that the bulk of research literature in community college neglects the study of what is happening *inside* mathematics classrooms: the students' experiences and the nature of instruction.

Table 2. Number of journal articles on the practice of mathematics teaching in community college, 2000–2017.

		Database	
Journal Title	Key Words in Abstract	ERIC	Education Research Complete
Community College Journal of Research and Practice	"mathematics"	51	0
New Directions for Community Colleges	"mathematics"	5	4
Journal of Developmental Education	"mathematics"	35	52
Mathematics Teacher Educator	"community college" or "two-year college"	0	0
Educational Studies in Mathematics	"community college" or "two-year college"	1	1
Mathematical Thinking and Learning	"community college" or "two-year college"	0	0
Journal of Mathematical Behavior	"community college" or "two-year college"	1	2

While we did find a few articles that illuminated the success of some community college students, success was often measured by course grades and/or pass rates. For example, there were several articles concerning *Pathways* and the general trend toward students taking fewer than three courses in the developmental mathematics sequence. However, course grades and pass rates constitute a very narrow definition of success, as these do not take into consideration any changes in the students' attitudes towards mathematics, perseverance, or ability to transfer concepts to new mathematical situations (Cox, 2016; Mesa, Wladis, et al., 2014).

To be certain that we found all relevant research, we then turned to other peer-reviewed mathematics education journals and searched for articles that focused on community college instruction. For instance, *Mathematics Teacher Educator* included no community college articles on pedagogical content knowledge. *Educational Studies in Mathematics* had only one article of interest tangentially related to MKT, described below. Likewise, *Mathematical Thinking and Learning* and *Journal of Mathematical Behavior* also provided very few community college studies.

Despite the lack of MKT-specific research in community college in journals, we did find a few researchers who sought to examine student success in developmental mathematics through the examination of instructional practices. These researchers may not have explicitly used the MKT framework, but we argue that the implementation of the pedagogy described in many of these studies would necessitate teachers' expertise in MKT. For example, Garrett's (2013) case study examined one developmental mathematics student's journey, from wrestling with connections between multiple representations of functions, to the instructor's subsequent in-class discussions with the student to facilitate his conceptual understanding. This example of in-the-moment classroom dialogue is a hallmark of effective MKT, and can serve as a springboard upon which we can build a better understanding of MKT in the developmental mathematics classroom.

Interestingly, some studies alluded to non-pedagogical aspects of effective mathematics instruction, and we highlight two such studies here. Galbraith and Jones (2006) chronicled one developmental mathematics instructor's changes in teaching philosophy and practices over a 3-year period. Essentially, the instructor acknowledged the unique needs of the population of developmental mathematics students and adjusted her teaching accordingly by attending to the students' cognitive, social, and emotional needs. By doing so, the instructor "employed all aspects [of communication] to tell learners that 'I care about you, and what you have to say is important'" (Galbraith & Jones, 2006, p. 26).

Another study – one that was unearthed from our search in *Educational Studies in Mathematics* – also speaks to non-pedagogical aspects of mathematics instruction. Wheeler and Montgomery (2009) analyzed community college students' beliefs about mathematics learning and found that not only were beliefs closely related to each student's prior educational experiences, but that "students viewed the teacher as the most important factor in mathematics learning regardless of their experiences" (p. 302), and this was true for all three categories of learners identified – Active, Skeptical, and Confident Learner. In fact, the authors note that positive views of the teacher correlated to positive beliefs about learning mathematics and confidence in one's ability to do mathematics; conversely, negative views of the teacher correlated to negative attitudes towards learning mathematics, math anxiety, and decreased achievement. Thus, Wheeler and Montgomery (2009) challenge us by asking the following: "What characteristics are students looking for in a *good* teacher? Is it instructional skill, emotional support, or some combination of affective and cognitive elements?" (*p*. 303).

These studies on effective community college mathematics instruction including non-pedagogical components, can inform the direction of future research in community college MKT. It is not compelling enough for us to say that further research is needed just because it hasn't been done previously. Rather, institutional factors and student demographics suggest we are working with something very different here, so there is potential to expand the field of knowledge of MKT.

The case for MKT research in community college

Why is this research important? Above all, passing rates and completion rates in development mathematics are abysmal. The Community College Research Center's (CRCC) analysis of 35 community colleges following 63,650 students in three-tier developmental mathematics sequences found that 52% of the students *left the track* at some point in the sequence (compared to 35% who did not pass or complete a course at some point in the sequence). Considering the small 2% who failed the eventual credit-bearing mathematics course, this means just 11% truly succeeded in earning college credit in mathematics (CCRC, 2014).[3] In light of the above, it is no wonder Anthony Bryk, President of the Carnegie Foundation for the Advancement of Teaching, uttered the chilling words, "Developmental mathematics courses represent the graveyard of dreams and aspirations." (Merseth, 2011, p. 32). MKT research, conceptualized for the unique population of students in developmental mathematics in community colleges, may help mitigate this situation.

Additionally, there is a need for faculty development in community college. Empirical studies in MKT and classroom research can equip faculty with mechanisms for effective teaching in developmental mathematics, much like what has already been done in elementary education. Research findings can then be disseminated to the next generation of community college mathematics teachers.

In sum, although the MKT-related research literature in elementary, secondary, and university settings provides a solid framework for further research, the students, content, and context of the community college setting affect teaching in profound ways. These factors support investigating MKT in community college with specific attention to the high-needs area of developmental mathematics.

Teachers open up: What is special about teaching developmental mathematics at the community college?

In this section, we discuss the results from interviews with 12 expert community college faculty about their experiences teaching developmental mathematics. We sought specialized information about mathematics teaching at the developmental level – classroom successes and challenges, examples of student learning, and anything additional that faculty were willing to share about classroom experiences at the community college. We homed in on *craft knowledge*, or "wisdom of practice … [which] encompasses the wealth of teaching information that very skilled practitioners have about their own practice" (Leinhardt, 1990, *p*. 18). We begin with some findings that we anticipated as classroom teachers ourselves, and then move to results that are augmented with theoretical work on relational caring and students' expressed needs.

Participants

We solicited 12 volunteers by way of two email groups through AMATYC's professional network – Research in Mathematics Education for Two-Year Colleges (RMETYC) and the Developmental Mathematics Committee (DMC). Using criterion sampling (Mertens, 2010), we specified that the participants needed to have experience teaching developmental mathematics in the community college and a commitment to student success. Although 15 individuals contacted us for an interview, we only selected the first 12 people due to an independent ethics committee maximum of 12 participants.

Below are salient characteristics about the interviewees – the number of years teaching, specific developmental courses taught, and professional involvement in educational reform at the developmental level (see Question 0, Appendix). All names are pseudonyms (Table 3).

Data collection and analysis

The interview format was chosen as the data collection instrument to provide rich, in-depth descriptions of mathematics teaching from the perspective of each individual (Seidman, 2006). The instrument we used (as well as our rationale for each question) can be found in the *Appendix: Questionnaire* (Creswell, 2009).

The researchers and participants agreed to a predetermined time to schedule a telephone interview. The interviews were semi-structured, based on participants' responses to the questions. The conversations were audio-recorded once consent was granted, and the data were saved and archived.

A phenomenological approach was used to analyze and make sense of the data (Bogdan & Biklen, 1992; Strauss & Corbin, 1990) and codes were developed by each researcher, who independently listened and documented important observations. Both analyses were carried out inductively, imposing no *a priori* conditions on the participants' utterances. Given the early stages of this analysis, the first author developed three preliminary codes with plans to make a second pass to elaborate and refine the codes. Similarly, the second author developed eight preliminary codes in the form of questions, and identified four emergent themes, all of which would be revisited with a second listen of the audio files. We anticipated that a subsequent round of coding would utilize *focused coding*, whereby the codes would be grouped into categories as needed (Saldana, 2009).

Table 3. Characteristics of community college teachers interviewed.

Name	Number of Years Teaching*	Courses Taught	Curricular/Pedagogical Reform
Anne	10 years	Arithmetic, Prealgebra, Basic Algebra	Math Literacy, Pre-Statistics, STEM Pathways
Brianna	18 years	Transfer-level and Developmental Mathematics	Math Literacy for six years
Cathy	7 years	Beginning and Intermediate Algebra	–
Donna	10 years	Developmental Mathematics, Math Literacy	Math Literacy
Erica	5 years	Arithmetic, Prealgebra, Basic Algebra, Intermediate Algebra,	Math Literacy
Frank	7 years	Basic Algebra, Intermediate Algebra, Support Courses for Algebra	Rethinking College Algebra
George	3 years	Arithmetic, Intermediate Algebra	Alternate Pathways, Pre-Statistics
Harold	20 years	Developmental Mathematics	–
Ingrid	20+years	Transfer-level and Developmental Mathematics	Curriculum Development
John	Nearly 20 years	Transfer-level and Developmental Mathematics	Alternate Pathways, STEM Pathways
Kim	Nearly 20 years	Basic Mathematics, Intermediate Algebra	–
Lisa	9 years	Developmental Mathematics, Elementary Algebra, Intermediate Algebra	–

For brevity purposes, *number of years teaching* includes years where faculty taught full- or part-time as well as experiences outside of the community college environment (e.g., university or high school teaching).

Preliminary findings

As evidenced in the survey instrument, we sought in the participants' descriptions of both mathematical and pedagogical experiences – perhaps preliminary references to MKT or SCK.[4] However, when we compared notes after the initial phase of data analysis, several commonalities appeared *nonmathematical* in nature. In fact, all 12 interviewees alluded to the importance of *caring* as embedded in their instructional practices.[5] While our main goal was (and still is) to generate theory through constant comparative method (Glaser, 1978; Strauss & Corbin, 1994, 1998), we believe the descriptions in their current form are worthy of dissemination with the research community. Thus, our intent is to formulate working hypotheses for what MKT might look like in the developmental setting but we feel it timely and worthwhile to report on the nonmathematical components that may be precursors to effective mathematics teaching at the community college.

At the start of the interview, we asked participants about the role of the community college (Question 1). Our intent here was to acclimate the participants to the interview format and allow the subjects to feel comfortable with answering questions. All participants answered this question with ease. Second, we asked participants to describe a "good day" in the classroom (Question 2); two-thirds mentioned student engagement, classroom discourse, and/or student questioning as evidence of successful teaching. In the same vein, "breaking away from tradition" was mentioned by half of the teachers as both a reason for their success in the classroom (Question 3) and as general advice to those just entering the profession (Question 5). When probed further, we found that "tradition" was defined as either lecturing to students or following a compartmentalized, skills-based curriculum. Nontraditional experiences included novel curricula (e.g., Carnegie *Pathways*), experiences with authentic information (e.g., using Census data), perplexing problem situations, and/or nontraditional pedagogy (e.g., inquiry-based learning). It is important to emphasize that *any* assortment of experiences from this list was deemed potentially effective because the traditional models proved otherwise. Faculty explained that many students were repeating courses (sometimes four to five times) under traditional delivery with the same outcome. Given these nontraditional experiences are outgrowths of community college reform movements, these responses made sense in this context.

With respect to students, teachers made several remarks pointing to challenges unrelated to mathematics. Student issues included indifferences to school, not completing assignments, and poor attendance. The teachers labeled these challenges as extrinsic barriers to helping students since they had less to do with mathematical readiness and more to do with broader issues of schooling. To add, some remarks suggested that students, irrespective of "good teaching," were unlikely to succeed. Brianna spoke of students failing courses because of not completing the work; this led to repeating courses, using up one's financial aid, and not progressing through the developmental sequence: "It's like having your bike in the wrong gear when you're going up a hill … people are passing you and you're like, 'What's wrong here?" Cathy added that students sometimes make poor decisions that are unrelated to mathematics. These were recurring challenges we heard from many of the teachers. In sum, the teachers' experiences suggest that many of the students cannot ask for help, are unaware of what they need and/or know, and cannot self-evaluate. This makes navigating the college experience extremely difficult. Unfortunately, the casualty is often mathematics.

On a positive note, seven of the 12 teachers mentioned the importance of believing in students. Donna exemplified how she thought differently of her students than many of her peers: "[my colleagues] will write off those students quickly, but they're really diamonds in the rough." Ingrid echoed this sentiment in that, "You need to keep your expectations high … .these students will absolutely amaze you." While the teachers we interviewed were keenly aware of the aforementioned indifferences that could infiltrate their classrooms, there was a profound sense that a culture of success could be built from expert management and interaction with students. Describing this often-impoverished group as "diamonds" and "amazing" suggest that the teachers know it is in their power to effect change in their classrooms.[6] We articulate what we suspect to be the key component of this change in the next section.

Does caring count?

In 1969, Mr. Fred Rodgers' gave a moving testimony before the Senate Commerce Committee in an effort to save public television from financial ruin. His pitch included the words, "I give an expression of care every day to each child." In this section, we discuss a surprisingly similar relationship between student and teacher in the context of developmental mathematics teaching. We draw on two emergent themes we believe make these 12 teachers effective, and subsequently, their students successful. First is the teachers' attention to caring for students and building a culture of mutual trust. This, of course, does not suggest that other teachers do not care for their students as people; simply put, what we heard from these teachers is the deepest sense of care, akin to a mother-child connection. Moreover, the teachers emphasized students' awareness of receiving this care, and reflexively, caring more about their academic progress. While establishing such bonds may be common at the elementary level, such discussions are less common in the college environment.

Second, in 11 of the 12 instances, the teachers described moments in instruction when students' needs usurped mathematical content. When we reference students' needs, we speak not of assumed needs such as needing a writing instrument but of students' *expressed needs* (e.g., regarding an emotional, personal, or moral dilemma). Through the lens of the work of Nel Noddings (1992, 2012), teachers who are attentive to students' expressed needs build stronger relationships than those who simply attend to students' assumed needs. Although Noddings's work emanates from children's needs and moral education – far removed from the subject-specific work of mathematics and developmental education – her attention to democratizing education and social justice suggest it is a useful framework with which to understand what teachers know about teaching developmental mathematics.

Caring and trust

> … there is a challenge to care in school. The structures of current schooling work against care, and at the same time, the need for care is perhaps greater than ever.
>
> (Noddings, 1992, p. 20)

The above quote typifies what we believe is a strong conviction of many of the teachers we interviewed. The caring we reference extends beyond that of a character attribute or any readily observable conduct. In the context of care ethics, the notion of *caring relation* clearly defines the roles of *carer* and *cared-for*. Even while many caring relations are not equal (e.g., doctor-patient or lawyer-client) they must be mutually receptive, for a caring relation cannot be established and sustained without continued maintenance of caring (Noddings, 1992). The role played by the cared-for is to signal that the care has been received; this validates the instance of caring offered and forms a *caring relation* (Noddings, 1999). Although one must display *virtue caring* in most aspects of daily living – a somewhat superficial expression of caring – we feel that many of the teachers were establishing an environment of *relational caring* as they tended to *both* the assumed and expressed needs of their students (Noddings, 1992, 1999). We provide evidence of these claims through quotes from our interviewees presently.

In describing her students, Brianna's words carry a maternal, empathic flavor in her instinct to understand her students and connect with them: "They know that I care about them and I think when you care, they care more." Her words extend beyond a friendly personality or students' willingness to negotiate in the game of school mathematics. She mentions an awareness that students are recipients of this care and that this alters student behavior as "they care more." Once a caring relation is established, the teacher has open access to helping students with personal challenges. It is worth noting that some students are reluctant to reveal deficiencies in their knowledge and this puts them in a vulnerable state which isn't always easy to share:

> Seeing the struggle that these students have and knowing that I [pause] can *help* them. That's just like an amazing feeling. And getting them to see that, they *can* do it … .I just love every single day. That just keeps me going back.

This, in a nutshell, is how Brianna described *mathematics* teaching.

Lisa also articulated the importance of caring, but added that students are in a position to accept or reject this offering. Through the lens of care ethics, this distinction is precisely what extends virtue caring to that of relational caring: "I think they [students] need to know that you [the teacher] care … .Trying to convince them to buy into what I'm doing – they don't, they don't do that for someone who doesn't care about them." The "buy-in" of which Lisa speaks underpins the caring relation and can extend to curricular reform and pedagogical innovation. For example, Ingrid affirmed changes in her teaching were based on the failures of a skills-based curriculum: "That curriculum is not built to make connections. We know it doesn't work." Half of the teachers interviewed mentioned curricular reform and pedagogical innovation as meeting the needs of their students. As teachers ourselves, we understand that sometimes coverage of topic A is a necessity to cover topic B and there may be little justification beyond this to provide thoughtful purpose for teaching topic A. This is commonplace at the lower levels of algebra. Noddings's (2005) words echo loudly here:

> Students will work on even trivial material for teachers they like and trust. Such teachers admit to their students that some subject matter is trivial and that, in a sense, we are all caught in a curriculum that offers both meaning and nonsense. The teacher's message is that we'll get through the nonsense together and work eagerly toward the construction of personal and collective meaning." (p. 155).

This is what student "buy in" looks like for these teachers.

Kim described her caring relation by way of reformulating how she teaches her classes. No stranger to the lecture model, she described changes she incorporated so that trust could be established as the basis for all mathematical learning. Having described her school's enrollment as predominantly Latinx,[7] Kim learned who her students were and in what environments they thrive; she learned of the importance of community, culture, and family. "Almost nothing they do is in isolation. They accomplish so much as a group." This led to pedagogical and assessment shifts in her practice: "I started giving group quizzes at the end of class since this is familiar to them – it builds community and trust in the educational process." The results of this innovation have meant a boost in her students' success and the relationships she has built. Kim feels she has taken a page out of her students' book; rather than requiring that students navigate a subject with the odds stacked against them, she has imported the mathematics into their way of living. John summarizes this best: "Care about your students as people – as individuals. Students will know if you're genuine."

Student before content

> When should teachers put aside the assumed need to learn a specific aspect of subject matter and address the expressed need of the student for emotional support, moral direction, or shared human interest?
>
> (Noddings, 2012, p. 772)

Two additional attributes of effective developmental mathematics teaching described by these teachers are shared here. The first of these attributes relates specifically to mathematical readiness. Teachers of developmental mathematics are willing to help students directly out of the gates – regardless of negative attitudes toward mathematics, a cynical reluctance to engage, and/or an expectation of failure. Two-thirds of the teachers mentioned something to the effect of "meeting students where they are" and the realization that a class of 35 students may very well contain 35 different "meeting points." Donna mentioned that it took her years to realize that "you need to start where they are" while Brianna genuinely voiced, "I work with what I got, not what I wish I had."

The above contrasts sharply with what was described by some of the interviewees' colleagues (who teach mostly upper-division mathematics courses).[8] For example, teaching developmental mathematics was described by Donna's colleagues as "time in the trenches." Donna added that teaching these courses, "to me … this is my challenge, my work – there's a purpose to what I'm doing." Anne expressed a similar sentiment, suggesting that teaching developmental mathematics uses an entirely different skill set from teaching transfer-level math courses. From her description, these skills appear to have less to do with subject matter and more to do with building relationships with her students: "People who teach math are really good at math and so they may have a difficult time breaking it down for students or identifying with what students might find difficult. These people should probably not be teaching developmental math." In Donna's case, knowledge of effective teaching goes beyond subject matter knowledge. While it certainly includes how to effectively work with specific mathematical content (as the research attests), the teachers here explained that *knowing their specific students* – e.g., how to help one overcome an academic or personal challenge – is equally important. While the research in MKT includes Knowledge of Content and Students (KCS) as a component, it sidesteps caring and relationship-building, which seem vital to the mathematics teaching described here.

The second attribute of effective teaching described by the teachers highlights the learning environment and a sense of belonging. Nearly half of those interviewed expressed the tension of focusing on their job of mathematics teacher but realizing that this goal was not realistic without first building trust with students. Frank discussed the apparent incongruity of teaching what is traditionally elementary school content (e.g., fractions) to adults: "It is difficult to create a collegial experience with this kind of mathematics and with each other. It takes a special kind of skill." In a similar way, George – a fairly inexperienced teacher – shared this view: "We need to teach students how to belong … practice the language of math in a safe nonjudgmental space, be understanding, kind, a good listener." From the above utterances, we see that the problems of education and personal fulfillment are entangled and it is nearly impossible to make progress in one arena without tending to the other (Noddings, 2005; Su, 2018). Teachers clearly expressed the need to establish a warm environment – one that is safe and inviting for learning. Once this setting is established, one can begin the task of teaching.[9] While this may seem like an undertaking separate from mathematics teaching, it explains why (a) some faculty refer to teaching developmental courses as "beneath" them, and (b) the inclusion of noncognitive factors has not been widely embraced in MKT research. Additionally, we recognize the need to broaden our findings to programs embracing co-requisite mathematics models in favor of the traditional developmental mathematics sequence. These co-requisite models often address nonmathematical needs such as study skills, metacognition, and social-emotional support, which is consistent with our findings here (Ackerman & Wilkerson, 2018).

The caring map

Mesa (2017) argues the following: "Researchers need to be informed by findings from K-12 and university settings, but they need to be conscious that there will more than likely be a need to reinterpret and redefine constructs to fit the community college context" (*p*. 962). While many assert that MKT can be generalized to K-12 (e.g., Hill, Ball, et al., 2008; Howell et al., 2016), can this model be applied to community college mathematics instruction at the developmental level? Given the results of our interviews, in which all 12 of the community college mathematics teachers embedded *caring* into their daily instruction, we propose a theoretical extension to the traditional MKT model (Ball et al., 2008).

Teaching is *relationally defined* and filled with *caring occasions* (Noddings, 1988). Noddings (2012) claims that teachers must balance content and caring by listening to students' needs, weaving care into the mathematics curriculum to ensure attendance to students' expressed needs. How does a teacher make time for this? "My answer is that establishing such a climate is not 'on top' of other things, it is *underneath* all we do as teachers" (Noddings, 2012, p. 777). This sentiment was echoed by all of our interviewees. Therefore, we propose that the domains of the MKT egg apply to

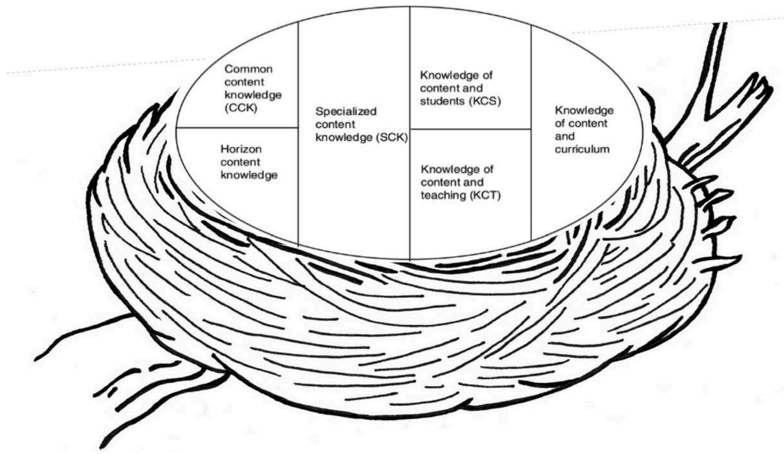

Figure 2. Ball et al. (2008) domains of MKT, with a foundational nest of caring, or the *Caring Map*.

developmental mathematics instruction in community college only if the egg sits in a foundational nest of caring, or the *Caring Map* (Figure 2).

At this point in our investigation, this extension of MKT theory is speculation, and further research is needed to determine the extent to which teachers' attention to caring and students' needs supports mathematics teaching and learning. But our conjectures align with those of Mesa (2017): constructs developed from K-12 research findings may need to be reinterpreted to fit the context of community college.

We recognize that this notion is not new; others have explored similar connections between mathematics instruction and teacher caring. The instructor in Galbraith and Jones (2006)'s three-year case study described how caring and relationship-building were necessary components of effective developmental mathematics instruction. Mesa, Celis, and Lande (2014) also allude to nonmathematical themes: "Instructors who privilege a student-support approach do not place students' mastering the content as the main goal of instruction. Rather, they focus on improving students' self-confidence and in developing relationships among students and between the students and instructor" (p. 135). Conversely, Jackson and Leffingwell (1999) found that insensitive, uncaring overt and covert behaviors of K-16 teachers have profound implications on their students' motivation, success, and anxiety in mathematics.

Not unlike Gutiérrez's (2013a) assertion that political knowledge for teaching is an extension of content knowledge, we assert that MKT in community college is a different type of knowledge for teaching – knowledge that merges with the theory of caring. This synthesis of MKT and care is sensible given the circumstances of the typical student enrolled in developmental mathematics – relegated to a noncredit-bearing math class, carrying baggage of negative past experiences, (sometimes) repeating the course, and being taught by an adjunct instructor with fewer supporting resources.

Given the community college population's academic and social needs, it follows that a classroom environment borne of care would be necessary for productive discourse to flourish with developmental mathematics students. The long-term implications of this research trajectory could be profound. Developmental education – though a huge piece of the community college pie – is not exclusively the responsibility of community colleges. Many four-year colleges and universities are struggling with similar stories of low persistence, completion, and success. Formal research in community college teaching using MKT and the Caring Map may provide meaningful direction and guidance to alter the course.

Conclusion

Mathematical Knowledge for Teaching (MKT) has transformed the landscape of educational scholarship at the K-12 and post-secondary levels. The latest developments are examining teachers' MKT as embedded in the active work of classroom instruction, and how teachers can be supported to see MKT as knowledge in practice (e.g., Ghousseini, 2017). Thus, while MKT continues to be a topic of interest in mathematics education, it remains unexamined at the nation's community colleges, and absent altogether from developmental mathematics teaching. We wonder if this omission is due to the less-than-glamorous aura of *developmental mathematics at the community college*. However, these students constitute our most vulnerable population. Accordingly, the mathematics education community has the responsibility to believe in students who do not believe in themselves. And we have the responsibility to research those problems in need of solutions.

Some scholars have expressed a sincere optimism for students who face the steepest uphill climb and ask that educators rethink the status quo (Goldrick-Rab, 2010, 2016; Su, 2018). We have heard a similar optimism from interviews with dedicated teachers from two-year colleges. These teachers are special in the ways they interact with students, colleagues, subject matter, and curricular demands – all within the constraints of the community college system. Yet, articulating *how* and *why* these teachers are special has not been of general interest to mathematics educators (Nabb & Murawska, 2016, 2018, in press). To date, it appears that MKT is a useful framework in which to study this phenomenon and suggest improvements related to community college mathematics teaching.

From a practitioner point of view, we understand that teachers strive to increase craft knowledge and improve their practice. Given the recent movements in the community college – in particular, the sweeping reforms in developmental mathematics and the new standards for two-year college mathematics teaching (American Mathematical Association of Two-Year Colleges, 2018) – it is fitting to examine mathematics teaching at the developmental level. The core content in developmental mathematics – arithmetic, fractions, proportional reasoning, and algebraic thinking – provides a critical foundation for advanced study in the mathematical sciences. Furthermore, research and practice inform us that mathematics does not stand in isolation; one's experiences, emotions, successes, and failures intertwine with the mathematics one learns.

Community colleges are beacons of opportunity for many of today's first-generation, historically marginalized, and economically disadvantaged students (Mesa, 2017). Mirroring the call for research-based instructional practices in grades 9–12 (National Council of Teachers of Mathematics, 2018), it is time for MKT to be examined in the context of our nation's *teaching* colleges. Increasing success rates in community college developmental mathematics has profound effects for society at large. Our preliminary interviews with 12 faculty at two-year colleges suggest important amendments to existing frameworks as we know them (e.g., Mesa, 2017; Speer et al., 2015). These early findings suggest that teachers in these classrooms place the human experience – *caring* and *relationship-building* – above any act of day-to-day teaching. While we recognize as teachers that building relationships and providing emotional support are problematic with large class sizes, Francis Su reminds us these teachers are embracing "human flourishing" (2018). These teachers self-identify as "having a special skill set" and they have convinced us that *caring for who your students are* and *believing in what they can become* are critical components of high quality mathematics instruction. It is time that we, as a research community, provide this area the attention it deserves, and shine a light on conceptualizing and advancing mathematics teaching at the community college.

Notes

1. Recent reports indicate a drop in traditional developmental mathematics enrollment (Burdman, 2018; CMBM, 2018) – welcoming news for the "trap" it has become for students who place early in the developmental mathematics sequence.
2. In recent years, the community college has gained traction as a legitimate area to investigate mathematics teaching, both empirically and through the Scholarship of Teaching and Learning (SoTL) (Givvin, Stigler, &

Thompson, 2011; Mesa, 2017; Mesa et al., 2014; Sitomer et al., 2012; Stigler, Givvin, & Thompson, 2010; Urbina-Lilback, 2016).

3. This is updated from Bailey, Jeong, and Cho (2010).
4. At this stage in our work, we were not expecting to collect evidence of MKT, given we were not observing teachers interacting with students and/or mathematical work.
5. We see a striking parallel to Francis Su's commentary on human flourishing – particularly to the components of justice, freedom, community, and love (Su, 2017, 2018).
6. We see these descriptions as analogous to Delpit's (2012) notion of *warm demanders*, who teach marginalized children of color.
7. Some researchers use the term "Latinx" as a decentering, gender-neutral alternative to the traditional Latino/a (Salinas & Lozano, 2017) or "Latin@" (Gutiérrez, 2013b; Salinas & Lozano, 2017) as inclusive of those who identify as LGBTQ.
8. These descriptions – coming directly from the 12 interview candidates – were heard in a variety of different settings including department meetings, casual meetings in hallways, etc.
9. It is important to note that we are not aware if this setting is established prior to teaching or if it co-emerges within the complex act of mathematics teaching. This, we feel, is one of many possible fruitful paths in which to examine MKT in community college developmental education.

References

Ackerman, J., & Wilkerson, D. (2018, February). Results from our math 146 co-requisite course redesign at Jefferson. *Annual Meeting of Kentucky Mathematical Association of Two Year Colleges*, Berea, KY.

American Mathematical Association of Two-Year Colleges. (2018). *AMATYC impact: Improving mathematical prowess and college teaching*. Memphis, TN: Author.

Bailey, T. (2009). Challenge and opportunity: Rethinking the role and function of developmental education in community college. *New Directions for Community Colleges*, *145*, 11–30. doi:10.1002/cc.352

Bailey, T., Jeong, D. W., & Cho, S. W. (2010). Referral, enrollment, and completion in developmental education sequences in community colleges. *Economics of Education Review*, *29*(2), 255–270. doi:10.1016/j.econedurev.2009.09.002

Ball, D. L. (1988). *Knowledge and reasoning in mathematical pedagogy: Examining what prospective teachers bring to teacher education. (Volumes I and II)* (Doctoral dissertation). Retrieved from http://www.ulib.niu.edu

Ball, D. L., Lubienski, S. T., & Mewborn, D. S. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of research on teaching* (4th ed., pp. 433–456). Washington, DC: American Educational Research Association.

Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching. *American Educator*, *29*(3), 14–23.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*(5), 389–407. doi:10.1177/0022487108324554

Bogdan, R., & Biklen, S. (1992). *Qualitative research for education: An introduction to theory and methods*. Boston, MA: Allyn & Bacon.

Boylan, H. R. (2011). Improving success in developmental mathematics: An interview with Paul Nolting. *Journal of Developmental Education*, *34*(3), 20–27.

Burdman, P. (2018). OPINION: Fewer students have to take college remedial math, data show. Retrieved from http://hechingerreport.org/college-remedial-math-enrollment-continues-to-drop-data-show/

Carpenter, T. P., Fennema, E., Peterson, P. L., & Carey, D. A. (1988). Teachers' pedagogical content knowledge of students' problem solving in elementary arithmetic. *Journal for Research in Mathematics Education*, *19*(5), 385–401. doi:10.2307/749173

Clark, B. R. (1960). The 'cooling-out' function in higher education. *American Journal of Sociology*, *65*, 569–576. doi:10.1086/222787

Clark, B. R. (1980). The 'cooling out' function revisited. *New Directions for Community Colleges*, *32*, 15–31. doi:10.1002/cc.36819803204

Community College Research Center. (2014, January). *What we know about developmental education outcomes*. New York, NY: Teachers College, Columbia University. Retrieved from http://ccrc.tc.columbia.edu/media/k2/attachments/what-we-know-about-developmental-education-outcomes.pdf

Conference Board of the Mathematical Sciences. (2018). *Statistical abstract of undergraduate programs in the mathematical sciences in the United States fall 2015 CBMS survey*. Retrieved from http://www.ams.org/profession/data/cbms-survey/cbms2015-Report.pdf

Cox, R. D. (2016, November). *Observing from the back of the classroom: Dilemmas of developmental math research*. Paper presented at the Research Session of the 42nd Annual Conference of the American Mathematical Association of Two-Year Colleges, Denver, CO.

Creswell, J. W. (2009). *Research design: Qualitative, quantitative, and mixed methods approaches* (3rd ed.). Thousand Oaks, CA: Sage Publications.
Delpit, L. (2012). *Multiplication is for white people: Raising expectations for other people's children*. New York, NY: The New Press.
Eisenhart, M., Borko, H., Underhill, R., Brown, C., Jones, D., & Agard, P. (1993). Conceptual knowledge falls through the cracks: Complexities of learning to teach mathematics for understanding. *Journal for Research in Mathematics Education*, *24*(1), 8–40. doi:10.2307/749384
Even, R. (1993). Subject-matter knowledge and pedagogical content knowledge: Prospective secondary teachers and the function concept. *Journal for Research in Mathematics Education*, *24*(2), 94–116. doi:10.2307/749215
Galbraith, M. W., & Jones, M. S. (2006). The art and science of teaching developmental mathematics: Building perspective through dialogue. *Journal of Developmental Education*, *37*(2), 2–33.
Garrett, L. (2013). Update: Flawed mathematical conceptualizations: Marlon's dilemma. *Journal of Developmental Education*, *30*(2), 20–27.
Ghousseini, H. (2017). Rehearsals of teaching and opportunities to learn mathematical knowledge for teaching. *Cognition and Instruction*, *35*(3), 188–211. doi:10.1080/07370008.2017.1323903
Givvin, K. B., Stigler, J. W., & Thompson, B. J. (2011). What community college developmental mathematics students understand about mathematics, part 2: The interviews. *MathAMATYC Educator*, *2*(3), 4–18.
Glaser, B. (1978). *Theoretical sensitivity: Advances in the methodology of grounded theory*. Mill Valley, CA: Sociology Press.
Goldrick-Rab, S. (2010). Challenges and opportunities for improving community college student success. *Review of Educational Research*, *80*(3), 437–469. doi:10.3102/0034654310370163
Goldrick-Rab, S. (2016). *Paying the price*. Chicago, IL: University of Chicago Press.
Gutiérrez, R. (2013a). Why (urban) mathematics teachers need political knowledge. *Journal of Urban Mathematics Education*, *6*(2), 7–19.
Gutiérrez, R. (2013b). The sociopolitical turn in mathematics education. *Journal for Research in Mathematics Education*, *44*(1), 37–68. doi:10.5951/jresematheduc.44.1.0037
Hill, H. C., Sleep, L., Lewis, J. M., & Ball, D. L. (2007). Assessing teachers' mathematical knowledge: What knowledge matters and what evidence counts? In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 111–155). Charlotte, NC: Information Age Publishing.
Hill, H. C., & Ball, D. L. (2004). Learning mathematics for teaching: Results from California's mathematics professional development institutes. *Journal for Research in Mathematics Education*, *35*(5), 330–351. doi:10.2307/30034819
Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, *39*(4), 372–400.
Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J., Phelps, G. C., Sleep, L., & Ball, D. L. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction*, *26*, 430–511. doi:10.1080/07370000802177235
Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, *42*(2), 371–406. doi:10.3102/00028312042002371
Hill, H. C., Schilling, S. G., & Ball, D. L. (2004). Developing measures of teachers' mathematics knowledge for teaching. *The Elementary School Journal*, *105*(1), 11–30. doi:10.1086/428763
Hoover, M., Mosvold, R., Ball, D. L., & Lai, Y. (2016). Making progress on mathematical knowledge for teaching. *The Mathematics Enthusiast*, *13*(1), 3–34.
Howell, H., Lai, Y., Phelps, G., & Croft, A. (2016). *Assessing mathematical knowledge for teaching beyond conventional mathematical knowledge: Do elementary models extend?* Paper presented at the annual meeting of the American Educational Research Association, Washington, DC.
Jackson, C. D., & Leffingwell, R. J. (1999). The role of instructors in creating math anxiety in students from kindergarten through college. *Mathematics Teacher*, *92*(7), 583–586.
Jennings, F. G. (1970). Junior colleges in America: The two-year stretch. *Change: the Magazine of Higher Learning*, *2*(2), 15–25. doi:10.1080/00091383.1970.10567857
Khakbaz, A. (2016). Mathematics university teachers' perception of pedagogical content knowledge. *International Journal of Mathematical Education in Science and Technology*, *47*(2), 185–196. doi:10.1080/0020739X.2015.1065453
Leinhardt, G. (1990). Capturing craft knowledge in teaching. *Educational Researcher*, *19*(2), 18–25. doi:10.3102/0013189X019002018
Manizade, A. G., & Mason, M. M. (2011). Using Delphi methodology to design assessments of teachers' pedagogical content knowledge. *Educational Studies in Mathematics*, *76*, 183–207. doi:10.1007/s10649-010-9276-z
McCrory, R., Floden, R. E., Ferrini-Mundy, J., Reckase, M., & Senk, S. (2012). Knowledge of algebra for teaching: A framework of knowledge and practices. *Journal for Research in Mathematics Education*, *43*(5), 584–615. doi:10.5951/jresematheduc.43.5.0584
Merseth, K. K. (2011). Update: Report on innovations in developmental mathematics—Moving mathematical graveyards. *Journal of Developmental Education*, *34*(3), 32–38.

Mertens, D. M. (2010). *Research and evaluation in education and psychology* (3rd ed.). Thousand Oaks, CA: Sage Publications.

Mesa, V. (2017). Mathematics education at U.S. public two-year colleges. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp. 949–967). Reston, VA: NCTM.

Mesa, V., Celis, S., & Lande, E. (2014). Teaching approaches of community college mathematics faculty: Do they relate to classroom practices? *American Educational Research Journal*, *52*, 117–151. doi:10.3102/0002831213505759

Mesa, V., Wladis, C., & Watkins, L. (2014). Research problems in community college mathematics education: Testing the boundaries of K-12 research. *Journal for Research in Mathematics Education*, *45*(2), 173–192. doi:10.5951/jresematheduc.45.2.0173

Nabb, K., & Murawska, J. (2016, November). *Mathematical knowledge for teaching developmental courses at the community college: An unexplored terrain*. Paper presented at the Forty-Second Annual Meeting of the American Mathematical Association of Two-Year Colleges (Research Session), Denver, CO.

Nabb, K., & Murawska, J. (2018, April). *Mathematical knowledge for teaching at the developmental level: Does caring count?* Paper presented at the Forty-Third Annual Meeting of the Illinois Mathematical Association of Community Colleges (IMACC), Monticello, Illinois.

Nabb, K., & Murawska, J. (in press). Where is mathematical knowledge for teaching at the community college? *MathAMATYC Educator*.

National Council of Teachers of Mathematics. (2018). *Catalyzing change in high school mathematics*. Reston, VA: NCTM.

Noddings, N. (1988). An ethic of caring and its implications for instructional arrangements. *American Journal of Education*, *96*(2), 215–230. doi:10.1086/443894

Noddings, N. (1992). *The challenge to care in schools*. New York, NY: Teachers College Press.

Noddings, N. (1999). Caring and competence. In G. Griffin (Ed.), *The education of teachers* (pp. 205–220). Chicago, IL: National Society for the Study of Education.

Noddings, N. (2005). Identifying and responding to needs in education. *Cambridge Journal of Education*, *35*(2), 147–159. doi:10.1080/03057640500146757

Noddings, N. (2012). The caring relation in teaching. *Oxford Review of Education*, *38*(6), 771–781. doi:10.1080/03054985.2012.745047

Phelps, G., & Howell, H. (2016). Assessing mathematical knowledge for teaching: The role of teaching context. *The Mathematics Enthusiast*, *13*(1), 52–70.

Philipp, R. A., Ambrose, R., Lamb, L. L. C., Sowder, J. T., Schappelle, B. P., Sowder, L., … Chauvot, J. (2007). Effects of early field experiences on the mathematical content knowledge and beliefs of prospective elementary school teachers: An experimental study. *Journal for Research in Mathematics Education*, *38*(5), 438–476.

Rasmussen, C., & Marrongelle, K. (2006). Pedagogical content tools: Integrating student reasoning and mathematics in instruction. *Journal for Research in Mathematics Education*, *37*(5), 388–420.

Saldana, J. (2009). *The coding manual for qualitative researchers*. Thousand Oaks, CA: Sage Publications.

Salinas, C., Jr, & Lozano, A. (2017). Mapping and recontextualizing the evolution of the term *Latinx*: An environmental scanning in higher education. *Journal of Latinos and Education*, 1–14. doi:10.1080/15348431.2017.1390464

Seidman, I. (2006). *Interviewing as qualitative research: A guide for researchers in education and the social sciences* (3rd ed.). New York, NY: Teachers College Press.

Selling, S. K., Garcia, N., & Ball, D. L. (2016). What does it take to develop assessments of mathematical knowledge for teaching?: Unpacking the mathematical work of teaching. *The Mathematics Enthusiast*, *13*(1), 35–51.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*(2), 4–14. doi:10.3102/0013189X015002004

Silverman, J., & Thompson, P. W. (2008). Toward a framework for the development of mathematical knowledge for teaching. *Journal of Mathematics Teacher Education*, *11*(6), 499–511. doi:10.1007/s10857-008-9089-5

Simon, M. A. (1993). Prospective elementary teachers' knowledge of division. *Journal for Research in Mathematics Education*, *24*(3), 233–254. doi:10.2307/749346

Sitomer, A., Ström, A., Mesa, V., Duranczyk, I., Nabb, K., Smith, J., & Yannotta, M. (2012). Moving from anecdote to evidence: A proposed research agenda in community college mathematics education. *MathAMATYC Educator*, *4*(1), 35–40.

Speer, N. M., King, K. D., & Howell, H. (2015). Definitions of mathematical knowledge for teaching: Using these constructs in research on secondary and college mathematics teachers. *Journal of Mathematics Teacher Education*, *18*, 105–122. doi:10.1007/s10857-014-9277-4

Speer, N. M., Smith, J. P., & Horvath, A. (2010). Collegiate mathematics teaching: An unexamined practice. *Journal of Mathematical Behavior*, *29*(2), 99–114. doi:10.1016/j.jmathb.2010.02.001

Speer, N. M., & Wagner, J. F. (2009). Knowledge needed by a teacher to provide analytic scaffolding during undergraduate mathematics classroom discussions. *Journal for Research in Mathematics Education*, *40*(5), 530–562.

Stigler, J. W., Givvin, K. B., & Thompson, B. J. (2010). What community college developmental mathematics students understand about mathematics. *MathAMATYC Educator*, *1*(3), 4–16.

Strauss, A., & Corbin, J. (1994). Grounded theory methodology: An overview. In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of qualitative research* (pp. 273–285). Thousand Oaks, CA: Sage.
Strauss, A., & Corbin, J. (1990). *Basics of qualitative research: Grounded theory procedures and techniques*. Newbury Park, CA: Sage.
Strauss, A., & Corbin, J. (1998). *Basics of qualitative research: Techniques and procedures for developing grounded theory* (2nd ed.). Thousand Oaks, CA: Sage.
Su, F. (2017). Mathematics for human flourishing. *American Mathematical Monthly*, *124*, 483–493. doi:10.4169/amer.math.monthly.124.6.483
Su, F. (2018, April). *Closing keynote: Mathematics for human flourishing*. Washington, DC: Annual Meeting of the National Council of Teachers of Mathematics.
Thompson, P. W. (2016). Researching mathematical meanings for teaching. In L. English & D. Kirshner (Eds.), *Handbook of international research in mathematics education* (pp. 435–461). London, UK: Taylor and Francis.
Tsay, J., Judd, A., Hauk, S., & Davis, M. K. (2011). Case study of a college mathematics instructor; patterns of classroom discourse. *Educational Studies in Mathematics*, *78*, 205–229. doi:10.1007/s10649-011-9323-4
Urbina-Lilback, R. N. (2016). Snapshots of equitable teaching in a highly diverse classroom. *Mathematics Teacher*, *110*(2), 126–132. doi:10.5951/mathteacher.110.2.0126
Wagner, J., Speer, N., & Rossa, B. (2007). Beyond mathematical content knowledge: A mathematician's knowledge needed for teaching an inquiry-oriented differential equations course. *Journal of Mathematical Behavior*, *26*, 247–266. doi:10.1016/j.jmathb.2007.09.002
Wheeler, D. L., & Montgomery, D. (2009). Community college students' views on learning mathematics in terms of their epistemological beliefs: A Q method study. *Educational Studies in Mathematics*, *72*(3), 289–306. doi:10.1007/s10649-009-9192-2
Wu, H. (2009). What's sophisticated about elementary mathematics? *American Educator*, *33*(3), 4–14.
Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*, *27*(4), 458–477. doi:10.2307/749877

Appendix: Questionnaire

Interview Question	Rationale
0. Can you tell me a little bit about your background teaching developmental mathematics at the community college?	*Focus*: Background information Experience in community college mathematics education, number of years teaching, areas of expertise, professional involvement in educational reform
1. You are a teacher at a community college. Some argue community colleges are an extension of high school, some argue it is the beginning of university/college work, others argue that neither description quite fits. Can you tell us your view on this?	*Focus*: The community college What makes the community college unique? (not specifically asking about mathematics nor teaching). Who enrolls? What attributes set them apart?
2. Think about a recent day in your math classroom that went particularly well. Describe what the students were doing and what you were doing. How did you know it was a good day?	*Focus*: Teaching What are the things you do and your students (need to) do to learn?
3. At many community colleges, dev-ed math success rates are very low. However, you have documented successes in many of your courses. To what do you attribute your effectiveness as a dev-ed math instructor?	*Focus*: Developmental level mathematics What makes you (the teacher) special? Do you do things that other (less successful?) teachers fail to do?
4. What do you most enjoy about teaching dev-ed math, and what do you least enjoy about teaching dev-ed math?	*Focus*: Specialized content knowledge about teaching Why are you so effective? What have you not yet mastered?
5. If you were mentoring a new teacher about to teach a dev-ed math course at your community college, what advice would you offer them for successful teaching?	*Focus*: Craft knowledge, professional wisdom, knowledge in professional development, self-reflection Is this Mathematical Knowledge for Teaching?